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Effect of the Smoothing Parameter on Nonparametric Improved Recursive Double Homogeneously Weighted Moving Average Control Chart

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ABSTRACT

The study investigates the effect of varying the power of the smoothing parameter (λ) in a Nonparametric Improved Recursive Double Homogeneously Weighted Moving Average control chart based on the Wilcoxon Signed-Rank statistic (NPIRDHWMA-WSR) for enhanced process monitoring. Specifically, the research examines the influence of smoothing parameter powers ranging from λ^4 to λ^8 on the detection capability of the chart. A Monte Carlo simulation with 5,000 iterations was conducted under different process distributions, including normal, contaminated normal, and heavy-tailed distributions, to evaluate the Average Run Length (ARL) performance of the proposed configurations. The results show that increasing the smoothing parameter power improves detection sensitivity for moderate process shifts, with the λ^6 configuration providing the best balance between rapid detection and stability while maintaining the desired in-control performance ($ARL_0 \approx 500$). Furthermore, the proposed chart demonstrates strong distributional robustness across all tested scenarios. The practical applicability of the method was validated using wine quality data, where the chart successfully detected process shifts in chloride measurements, identifying out-of-control signals beginning around samples 18–19 and producing 22–23 alarms within 40 observations. Overall, the NPIRDHWMA-WSR control chart provides a flexible and effective tool for monitoring industrial processes under non-normal conditions, offering practitioners an evidence-based approach for selecting smoothing parameter configurations that balance detection speed and false alarm stability.

Keywords: Nonparametric control chart, NPIRDHWMA, Wilcoxon Signed-Rank, ARL, process monitoring, distributional robustness.

1.0 INTRODUCTION

Statistical Process Control (SPC) is an important concept that brings about stability in the quality of products and processes, as well as efficiency in operations in both manufacturing and service industries. One of the most basic tools in SPC, the control chart has been one of the most basic since its introduction in 1931 by Shewhart, the chart offers a framework of systematic identification of assignable causes of variation and process reliability. Control charts are used to identify and rectify abnormal process behavior by identifying random and special causes of variation before it resulted in the quality decline or inefficiency in production.

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The evolution of the control charts has been significant because, whereas the earlier models of control charts were only monitoring models and not dynamic process models, the new models have evolved to be memory-based and nonparametric models that have contributed to capturing the dynamics of the processes. Shewhart-type charts that were introduced early were useful in identifying big changes, but not sensitive enough to identify subtle changes in processes. This was followed by the introduction of CUSUM (Page, 1954) and EWMA (Roberts, 1959), which marked a major improvement through the incorporation of memory structures that used cumulative information or exponentially weighted information of the previous samples. These charts provided analytical tractability and best detecting characteristics on small shifts of processes, but had limitations in that they relied on the assumption of normality. Although the success of these methods depends on the assumption of normally distributed process data, their work is largely dependent on this assumption. Non-normal features are common in real-world data, e.g., skewness, heavy tails, or being contaminated by outliers, which lead to unreliable performance and exaggerated false alarm rates. Moreover, the small sample sizes, measurement errors, and non-linear dynamics are other factors that put the applicability of traditional parametric control charts to the test. These restrictions have triggered the evolution of nonparametric control charts that can sustain statistical validity and strength in a wide range of process conditions.

Control charts that use nonparametric statistics do not rely on any specific distributional assumptions and therefore can be used on processes with unknown or variable distributions. These are usually using rank-based statistics, especially the Wilcoxon rank-sum (WSR) and signed tests, which are less sensitive to normality violations and contamination. The integration of rank-based statistics enhances robustness under non-normal distributions, consistent with studies by Chakraborti (2011) and Qiu (2013), thus increasing its usability to real industrial processes. In line with such developments, scientists have remained at the forefront to perfect memory-type charts by enhancing their weighting mechanisms. The HWMA chart presented by Castagliola and Khoo (2017) was able to provide uniform weighting, which provides better control limits and better detection than EWMA charts. These advances have performed better in detecting the small shift and robustness than in previous memory-type charts. The improved detection performance of the proposed chart is consistent with previous findings on memory-type control charts such as HWMA and EWMA.

Other more recent studies have merged nonparametric methods with HWMA-type models to realize strength and delicacy in one charting model. Raza et al. (2024) created the HWMA chart based on the Wilcoxon rank-sum (WSR), combining rank statistics with the homogeneous weighting to be applied successfully in the non-normal distributions. The results extend the work of Abid et al. (2017) and

Rasheed et al. (2022) by demonstrating that higher-order smoothing structures can further improve sensitivity without compromising in-control performance. Oladipupo et al. (2025) enhanced the Nonparametric Double Homogeneously Weighted Moving Average (NPIRDHWMA) Control Chart to Track Process Location Changes. But the higher the value of lambda, the more unstable the charts could be, and the more inconsistent the control limits could be, as well as the high false alarm rates. This means that sophisticated recursive and double-weighting schemes are required to ensure consistency in performance between high 0 configurations.

Many existing memory-type control charts (EWMA, HWMA, EHWMA) assume normality, limiting their reliability for skewed or contaminated process data. The effect of higher powers of the smoothing parameter (λ) in recursive HWMA-type nonparametric charts has not been systematically investigated, particularly regarding stability and false alarm behavior. Such a lack of knowledge is the driving force behind the current research to study the possibility of having recursive double-weighting and rank-based mechanisms to improve the performance of control charts without swelling false alarms. The reasons behind this study are that there is a necessity to develop an SPC model which is stable, flexible and strong to a large variety of process distributions, particularly when the prevalent conditions are large λ. In order to overcome these issues, a Nonparametric Improved Recursive Double Homogeneously Weighted Moving Average (NPIRDHWMA) chart is suggested.

The purposes of this research are to: Build the improved variant of the NPIRDHWMA-WSR control chart with higher sensitivity to the robustness with raising the power of smoothing parameter λ from 5 to 8; To compare and contrast the performance of the proposed NPIRDHWMA-WSR (λ^6-λ^8) with the traditional NPIRDHWMA-WSR (λ^5) on different process distribution and shift magnitude and to confirm the practicality of the improved chart in the real industrial data, namely the Wine Quality data. The research objectives will help to contribute to a strong and computationally efficient SPC framework that is applicable in contemporary industrial surveillance settings. Additions to Knowledge: Introduction of a superior, doubled, homogeneously weighted moving average control chart based on Wilcoxon Signed Rank statistics and high power of smoothing to enhance detection ability. The simulation results obtained through empirical studies on the proven faster (up to 35-60% faster) and more consistent NPIRDHWMA-WSR in identifying small to moderate process changes compared to previous models. Assurance of strength under symmetric and asymmetric, contaminated and heavy-tailed process distributions, extending the usefulness of the chart when compared to more conventional parametric methods. Showing on a real food quality dataset with its flexibility and effectiveness in practice-oriented monitoring of processes.

The history of SPC techniques is an ongoing attempt to make processes of monitoring more robust and sensitive. Since the earliest model of Shewhart is based on memory-type and nonparametric hybrid designs, the sphere has evolved to more adaptive and distributionally robust methods. The proposed NPIRDHWMA chart is one of the important contributions in this further development as it incorporates the recursive and double-weighting methods combined with nonparametric ranking approaches. The combination guarantees better performance in the non-normality and stability in the various process environments, making NPIRDHWMA a potential tool in the next-generation quality control system.

The rest of this paper is organized in the following way. Section 2, competing and proposed control charts, is the description of the methodology and the formulation of the model. Section 3 proposal to implement the proposed control chart and simulation design. Section 4 is the results, analysis, and discussion. Section 5 is the illustrates example. Section 6: Summary, conclusions and future recommendations.

2.0 Competing and Proposed Control Charts.

This part would feature a comparative analysis of two non-parametric scheme of control charts, and how various powers of the smoothing factor (λ) affect the performance of the chart. The hypothesized NPIRDHWMA-WSR (Non-Parametric Improved Robust Double Homogeneously Weighted Moving Average Control Chart using the Wilcoxon Signed rank statistic) is evaluated using λ applied to the power of 5 to 8. To have a strict comparison, the chart provided by Abid et al. original chart-NPDHWMA-RSS is applied with the value of 4 as the power of λ. In the case of benchmarking, the NPIRDHWMA-WSR chart that was created by Oladipupo et al. (2025) is used with λ increased to the fifth power. The control chart structure in both the control charts uses the Wilcoxon Signed Rank statistic as its central monitoring statistic in identifying process location change.

$$\mathrm{WSR_t} = \sum_{j=1}^{n}\sum_{h=1}^{m} \mathrm{sign}(X_{tj(h)} - \theta_0)R^{+}_{tj(h)} \qquad (1)$$

where, θ_0 = Process Median, t = number of samples, j = number of observations and h = number of cycles

The statistic adopted in Equation (1) has a mean and variance of E(WSRt) = 0 and Var(WSRt) = $r(2r+1)(r+1)/6)\omega_0^2$, respectively (Oladipupo et al., 2025). A significant variable used to increase the sensitivity of the suggested chart is ω_0^2 or $-\xi_0^2$ = 0.352, provided by Abid et al. (2017), when n = 10. In practice, the $-\xi_0^2$ = 0.352 can be used to create control limits that better capture the actual process behavior, thus allowing the quality control practitioners to better detect changes in the processes and initiate timely corrective measures to maintain product quality and customer satisfaction.

2.1 Non-Parametric Double Homogeneous Weighted Moving Average with Ranked Set Sampling (NPDHWMA RS Control Chart).

Abid et al. (2017) state that the Non-Parametric Double Homogenous Weighted Moving Average with Ranked Set Sampling (NPDHWMA RSS) Control Chart control chart statistic is as follows:

$$\mathrm{NPDH_t} = \lambda^2\mathrm{WSR_t} + (1 - \lambda^2)\overline{WRS}_{t-1} \quad (3)$$

when t = 0, the value of NPDH = 0

The NPDHWMA-WSR control chart's control limits, according to Rasheed et al. (2022) are:

When t = 1

$$UCL_{(NPDHWMA_{RSS})t} = \mu_o + L\sqrt{\lambda^4 (r(2r+1)(r+1)/6)\omega_0^2}$$
$$CL_{(NPDHWMA_{RSS})t} = \mu_0 \quad (3)$$
$$LCL_{(NPDHWMA_{RSS})t} = \mu_o - L\sqrt{\lambda^4 (r(2r+1)(r+1)/6)\omega_0^2}$$

When t > 1

$$UCL_{(NPDHWMA_{RSS})t} = \mu_o + L\sqrt{\lambda^4 + ((1-\lambda)^2(1+\lambda)^2/t - 1)\left((r(r+1)(2r+1)/6)\right)\omega_0^2}$$

$$CL_{(NPDHWMA_{RSS})t} = \mu_o \quad (4)$$

$$LCL_{(NPDHWMA_{RSS})t} = \mu_o - L\sqrt{\lambda^4 + ((1-\lambda)^2(1+\lambda)^2/t - 1)\left((r(r+1)(2r+1)/6)\right)\omega_0^2}$$

A decision of out-of -control (OOC) is taken if NPDHt > UC 〖"L " 〗_("(NPDHWM" "A" _"RSS" ") t") or "NPD" "H" _"t" " < LC" "L" _(" (NPDHWM" "A" _"RSS" ")t,")

2.2 Improved nonparametric double homogenously weighted moving average, with Wilcoxon signed-rank (NPIRDHWMA-WSR) Control Chart.

The use of the non-parametric Wilcoxon signed-rank was used to track process shift with an improved Non-Parametric Double Exponentially weighted Moving Average using signed ranks Control chart (Oladipupo et al., 2025). It uses the double exponentially weighted moving average method to give weights to past observations. Using the Wilcoxon Signed Rank test, this chart has high resilience to non-normality, as well as gives strong possibilities to detect changes in the process. Abid et al. (2017) and Rasheed et al. (2022) suggest it to be a better performing process location shift detection compared with the Non-Parametric Double Homogeneous Weighted Moving Average with Ranked Set Sampling (NPDHWMA_RSS) Control Chart control chart statistic. The statistical parameters of the construction of the NPIRDEWMA-WSR control chart are outlined below:

$$NPIRDH_t = \lambda^2 WSR_t + (1 - \lambda^2)\overline{WRS}_{t-1}$$
$$NPIRDH_0 = 0$$

E(〖WSR〗 _t) = 0, and

$$Var\,(WSR_t) = \begin{cases} \lambda^5 (r(2r+1)(r+1)/6)\xi^2{}_0, & t = 1 \\ \lambda^5 + ((1-\lambda)^2(1+\lambda)^2/t - 1)\left((r(r+1)(2r+1)/6)\right)\xi^2{}_0, & t > 1 \end{cases}$$

Where λ ϵ (0,1] is a smoothing constant.

The NPIRDHWMA-WSR chart reveals a superior level of control limits to the chart, which increases the sensitivity of the chart to process changes. Based on the values of E(WSR_t) and Var(WSR_t) the control limits of the proposed chart will be:

When t = 1

$$UCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o + L\sqrt{\lambda^5\left((r(r+1)(2r+1)/6)\right)\xi^2{}_0}$$
$$CL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o \quad (5)$$
$$LCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o - L\sqrt{\lambda^5\left((r(r+1)(2r+1)/6)\right)\xi^2{}_0}$$

When t > 1

$$UCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o + L\sqrt{\lambda^5 + ((1-\lambda)^2(1+\lambda)^2/t - 1)\left((r(r+1)(2r+1)/6)\right)\xi^2{}_0}$$
$$CL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o \quad (6)$$
$$LCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o - L\sqrt{\lambda^5 + ((1-\lambda)^2(1+\lambda)^2/t - 1)\left((r(r+1)(2r+1)/6)\right)\xi^2{}_0}$$

If $NPIRDH_t > UCL_{(NPIRDHWMA\text{-}WSR)t}$ or $< LCL_{(NPIRDHWMA\text{-}WSR)t}$

“the underpinning process is OOC; otherwise, it is IC.”

2.3 Enhanced nonparametric improved recursive double homogeneously weighted moving average (ENPIRDHWMA -WSR) control chart.

Following this scheme, the NPIRDHWMA-WSR chart adds an adjustable power of lambda in the construction of its control limits. In this example, the power of λ is increased to 6, 7, and 8. In order to make the proposed chart more sensitive, we boost the strength of the smoothing constant, investigating powers p = 5 – 8 provides a systematic way to determine an optimal balance between rapid shift detection and in-control stability. Therefore, the sensitivity of the proposed chart was robust due to the introduction of λ (smoothing constant) to the variance of WSR_t to the existing model. Improved Nonparametric ENPIRDHWMA -WSR control chart with fixed expressions of variance and tuning advice. Perform a Monte Carlo simulation between ARL (zero-state and steady-state) and competent charts. The ENPIRDHWMA -WSR chart will be based on the NPIRDHWMA-WSR chart suggested by Oladipupo et al. (2025), but it will provide its own modification to the control limits that will make the chart more sensitive to process changes. The control limits to the proposed chart are as shown using the values of E (WSR_t) and Var (WSR_t) as follows:

$$ENPIRDH_t = \lambda WSR_t + (1 - \lambda^2)\bar{WSR}_{t-1}$$
$$\text{with } ENPIRDH_0 = 0$$

The control limits are defined as:

When t = 1

$$UCL_{(ENPIRDHWMA\text{-}WSR)t} = \mu_0 + L\sqrt{\lambda^p.\frac{r(r+1)(2r+1)}{6}\xi^2{}_0}$$
$$CL_{(ENPIRDHWMA\text{-}WSR)t} = \mu_o \quad (7)$$
$$LCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_0 - L\sqrt{\lambda^p.\frac{r(r+1)(2r+1)}{6}\xi^2{}_0}$$

If t > 1

$$UCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o + L\sqrt{\frac{(\lambda^p + (1-\lambda)^2\,(1+\lambda)^2)}{t-1}\left(\frac{r(r+1)(2r+1)}{6}\xi^2{}_0\right)}$$
$$CL_{(ENPIRDHWMA\text{-}WSR)t} = \mu_o \quad (8)$$
$$LCL_{(NPIRDHWMA\text{-}WSR)t} = \mu_o - L\sqrt{\frac{(\lambda^p + (1-\lambda)^2\,(1+\lambda)^2)}{t-1}\left(\frac{r(r+1)(2r+1)}{6}\xi^2{}_0\right)}$$

Similar to the competing chart, the process is marked out of control when $NPIRDH_t$ is out of the calculated limits. This chart allows a more flexible sensitivity adjustment by changing the exponent, allowing for a stronger performance assessment and tuning in real-life situations.

3.0 Enactment of the Proposed Control Chart.

This subtopic includes the description of the implementation and the evaluation of performance of the suggested NPIRDHWMA-WSR (Non-Parametric Improved Robust Double Homogeneously Weighted Moving Average Control Chart using Wilcoxon Signed Rank statistic). The experiment conducts a stringent evaluation of the performance of the chart in in-control (IC) and out of control (OOC) conditions. The performance analysis is based on the capability of the chart to identify changes in the location of the process in terms of the concepts of an average run length (ARL) and the related measures of variations as the key metrics of performance. These measures are critical in the determination of sensitivity, reliability, and the strength of the scheme of monitoring in general.

3.1 Performance Metrics

The main parameter of a control chart performance is the Average Run Length (ARL). It is the number of samples one expects to take before the initial out-of-control signal. ARL can be divided into two categories, ARL_0 (In-Control Average Run Length): The average number of samples, when a process is stable. The larger the value of ARL_0 the better, as it implies a reduced false alarm rate.

ARL_1 (Out-of-Control Average Run Length): It is proportional to the average number of samples needed to be detected to change in the true process. Smaller values of ARL_1 are desirable, which indicates that assignable causes are detected faster. In this analysis, nominal ARL_0 is taken as 500, and a sample size n = 10 is taken to provide a consistent reference point on comparative analysis. In order to have ARL, two secondary statistics on Standard Deviation of Run Length (SDRL) and Median Run Length (MDRL) are calculated to determine how irregularly (and how regularly) detection times vary.

3.2 Simulation Protocol

The effectiveness of the proposed NPIRDHWMA-WSR chart is assessed with the help of large Monte Carlo simulations performed in R and with 5,000 independent executions of each scenario. The following distributions are said to be robust to various data behaviors: Standard Normal distribution: N(0,1); Student t - distribution: (heavy-tailed, t(ν)), where heavy-tailed data are being studied, Contaminated Normal distribution: CN = (mixture of N(0,σ) and $N(0,\sigma^2)$), where mixtures of data and outliers are studied.

All distributions are homogenized to mean (or median) 0 and variance 1. In the case of OOC evaluation, process shifts are introduced in an orderly manner, and the respective run lengths are noted.

Monte Carlo Simulation Algorithm:

Algorithm: Monte Carlo Simulation for Run Length Evaluation

1. Specify model parameters
    - Sample size n = 10
    - Target in-control ARL_0 = 500
    - Smoothing parameter λ = 0.10
    - Lambda powers p = 4,5,6,7,8
    - Number of Monte Carlo repetitions = 5000.

2. Generate observations
    - Draw random samples from the specified distribution (Normal, Student-t, or Contaminated Normal).
    - Standardize the distribution so that the in-control process median equals $\theta_0 = 0$.

3. Compute the Wilcoxon Signed Rank statistic

$$WSR_t = \sum_{j=1}^{n}\sum_{h=1}^{m} \operatorname{sign}(X_{tj(h)} - \theta_0)R^{+}_{tj(h)}$$

4. Compute the chart statistic
    - Calculate the monitoring statistic for the competing and proposed charts using their respective updating formulas.

5. Determine control limits

- Calculate upper and lower control limits using the theoretical mean and variance expressions.

6. Evaluate run length
   - Record the sample index t when the chart statistic first exceeds the control limits.
7. Repeat simulation
   - Repeat steps 2–6 for 5000 independent runs.
8. Compute performance measure
   - Estimate ARL, SDRL, and MDRL across all runs.

3.3 Performance of In-Control and Out-of-Control Robustness.

A summary of the run length of NPIRDHWMA-WSR control chart with IC and OOC conditions under various symmetric and asymmetric distributions has been summarized in Table 1. All the parameters in the charts are adopted in accordance with those of the earlier foundation research to provide consistency and comparative results. Some of the important findings are as follows: ARL_0 Performance: The obtained ARL_0 values are always near the nominal target of 500, which proves the stability of the chart and low false alarm; ARL_1 Behavior: ARL_1 values decrease with more robust λ and bigger L and indicate the improved sensitivity and more rapid detection of shifts; Distributional Robustness: It is observed that the proposed chart is stable with different normal, heavy-tailed, and contaminated distribution cases and shows the low false alarm; SDRL Patterns: It is observed that

The control chart of the NPIRDHWMA-WSR is very robust and flexible to different process conditions. It has great In-Control stability and has a high Out-of-Control detection ability. Its performance in both Gaussian and non-Gaussian backgrounds has confirmed its applicability in the industrial world of today, where data contamination and distributional nonconformance are the norm.

4.0 Results and Discussion

This section gives and discusses the Run Length (RL) performance properties of the proposed Enhanced Non-Parametric Improved Robust Double Homogeneously Weighted Moving Average Control Chart (ENPIRDHWMA-WSR) with λ^6 and its base NPIRDHWMA-WSR of λ^5. Comparison was done under normal distribution, Contaminated normal distribution (CN), and t(4) distribution, with nominal in-control Average Run Length (ARL_0) of 500 and sample size n = 10. Table 1 and Table 5 show the summary of the simulation results that imply the ARL, Standard Deviation of Run Length (SDRL), and Median Run Length (MDRL) values to a wide scale of process shifts (0.025 - 5.0). It aims at assessing the efficiency of charts, sensitivity to small and moderate changes, and resistance to non-normality.

TABLE 1: RL characteristics of the competing NPDHWMA-RSS control chart under different distributions with nominal ARL0 = 500, and n = 10

(λ, L)	Distr	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05 ,1.064	Normal	ARL	295.32	278.04	262.56	247.59	178.62	106.25	59.98	29.30	3.70	1.06	1.00	1.00	1.00
		MDRL	295.00	278.00	263.00	248.00	181.00	114.00	76.00	50.00	1.00	1.00	1.00	1.00	1.00
		SDRL	25.57	26.95	24.76	24.75	25.79	30.35	32.46	27.58	9.22	1.19	0.00	0.00	0.00
	CN	ARL	77.57	74.01	71.95	67.36	50.93	26.22	10.49	3.48	1.07	1.00	1.00	1.00	1.00
		MDRL	86.00	84.00	82.00	79.00	67.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	27.57	28.40	27.81	29.64	30.08	26.89	18.08	9.23	1.44	0.00	0.00	0.00	0.00
	t(4)	ARL	138.04	133.42	129.57	125.00	100.57	69.47	42.77	22.46	3.73	1.12	1.00	1.00	1.00
		MDRL	143.00	139.00	135.00	131.00	110.00	83.00	62.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	31.79	31.35	31.02	32.22	33.91	32.64	30.72	24.91	9.23	1.73	0.00	0.00	0.00
0.1 , 1.306	Normal	ARL	175.13	162.76	150.86	140.69	95.47	55.50	33.01	17.82	3.30	1.06	1.00	1.00	1.00
		MDRL	173.00	161.00	149.00	139.00	95.00	57.00	37.00	25.00	1.00	1.00	1.00	1.00	1.00
		SDRL	25.77	23.82	21.60	19.40	13.73	11.30	12.73	12.51	5.58	0.84	0.00	0.00	0.00
	CN	ARL	40.89	39.31	38.15	36.64	28.86	17.16	8.65	3.59	1.06	1.00	1.00	1.00	1.00
		MDRL	42.00	41.00	40.00	39.00	33.00	25.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	8.75	9.29	9.24	9.65	11.34	12.23	9.97	6.14	0.92	0.00	0.00	0.00	0.00
	t(4)	ARL	72.67	70.33	67.64	65.83	53.34	36.83	24.29	14.36	3.08	1.14	1.00	1.00	1.00
		MDRL	73.00	71.00	69.00	67.00	55.00	41.00	30.00	22.00	1.00	1.00	1.00	1.00	1.00
		SDRL	12.63	12.99	13.27	12.42	12.46	13.22	13.13	11.78	5.34	1.32	0.00	0.00	0.00
0.25 , 2.113	Normal	ARL	275.67	236.52	208.57	180.34	86.98	35.24	19.22	11.85	3.97	1.22	1.00	1.00	1.00
		MDRL	262.00	227.00	200.00	173.00	83.00	34.00	19.00	13.00	1.00	1.00	1.00	1.00	1.00
		SDRL	97.53	82.36	73.23	61.85	27.12	8.45	4.23	3.93	3.39	1.03	0.10	0.00	0.00
	CN	ARL	23.03	22.00	21.11	20.47	16.48	11.90	8.41	5.26	1.57	1.01	1.00	1.00	1.00
		MDRL	23.00	22.00	21.00	20.00	16.00	12.00	10.00	8.00	1.00	1.00	1.00	1.00	1.00
		SDRL	3.68	3.41	3.14	3.16	2.61	2.83	3.51	3.66	1.66	0.24	0.08	0.00	0.00
	t(4)	ARL	52.55	49.56	47.44	44.54	33.62	21.80	14.83	10.20	3.82	1.38	1.01	1.00	1.00
		MDRL	50.00	48.00	46.00	43.00	32.00	21.00	15.00	11.00	1.00	1.00	1.00	1.00	1.00
		SDRL	15.37	14.24	13.33	12.20	8.67	5.08	4.01	3.92	3.34	1.35	0.23	0.00	0.00
0.5 , 2.376	Normal	ARL	496.66	494.36	491.81	486.13	347.08	48.46	14.56	7.08	2.55	1.16	1.00	1.00	1.00

		MDRL	500.00	500.00	500.00	500.00	395.00	41.00	13.00	7.00	3.00	1.00	1.00	1.00	1.00
		SDRL	30.63	39.55	47.78	62.17	159.90	29.72	6.45	2.66	1.54	0.56	0.08	0.00	0.00
0.5 , 2.376	CN	ARL	23.27	20.98	19.15	17.68	11.39	6.97	4.77	3.29	1.38	1.02	1.00	1.00	1.00
		MDRL	21.00	19.00	18.00	16.00	11.00	7.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00
		SDRL	10.41	8.92	7.85	7.21	3.83	1.96	1.51	1.55	0.85	0.20	0.00	0.00	0.00
	t(4)	ARL	102.21	93.63	82.36	73.12	40.26	17.61	9.53	5.98	2.54	1.25	1.02	1.00	1.00
		MDRL	86.00	78.00	69.00	62.00	36.00	16.00	9.00	6.00	3.00	1.00	1.00	1.00	1.00
		SDRL	70.12	63.92	54.19	47.09	23.03	8.28	3.71	2.22	1.54	0.68	0.18	0.04	0.00

TABLE 2: RL characteristics of the NPIRDHWMA-WSR control chart under different distributions with nominal ARL0 = 500, n = 10 and λ5

(λ, L)	Distr	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05 ,1.064	Normal	ARL	237.07	216.93	201.58	182.07	112.79	46.43	17.29	5.13	1.14	1.00	1.00	1.00	1.00
		MDRL	288.00	270.00	255.00	240.00	170.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	119.96	116.77	112.80	111.37	88.19	55.90	31.49	14.56	2.17	0.00	0.00	0.00	0.00
	CN	ARL	15.34	14.09	13.21	11.11	6.36	2.23	1.25	1.04	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	32.24	30.52	29.22	26.52	18.36	8.09	3.33	1.19	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	82.34	79.46	72.69	69.35	49.19	23.41	10.06	4.05	1.13	1.00	1.00	1.00	1.00
		MDRL	131.50	128.00	120.00	116.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	71.61	69.85	68.16	66.15	55.33	37.33	22.48	12.05	2.12	0.00	0.00	0.00	0.00
0.1 , 1.306	Normal	ARL	158.73	144.60	134.32	122.98	76.31	33.04	13.38	4.87	1.15	1.00	1.00	1.00	1.00
		MDRL	170.00	157.00	146.00	135.00	92.00	51.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	55.61	53.79	50.76	48.60	39.68	28.34	17.60	9.35	1.56	0.00	0.00	0.00	0.00
	CN	ARL	14.71	14.20	12.90	11.88	7.01	2.59	1.30	1.05	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	19.78	19.13	18.21	17.39	12.71	6.19	2.48	0.94	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	52.70	49.47	47.87	45.15	33.13	17.94	8.28	3.75	1.17	1.01	1.00	1.00	1.00
		MDRL	69.00	66.00	64.00	62.00	49.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	34.40	33.63	32.65	31.90	27.51	20.36	13.17	7.64	1.66	0.26	0.00	0.00	0.00
0.25 , 2.113	Normal	ARL	182.10	163.11	146.82	131.90	73.13	31.13	15.12	7.12	1.55	1.01	1.00	1.00	1.00

		MDRL	178.00	159.00	143.00	129.00	72.00	32.00	18.00	10.00	1.00	1.00	1.00	1.00	1.00
		SDRL	54.21	48.24	42.18	38.84	21.72	10.58	8.07	6.10	1.89	0.17	0.00	0.00	0.00
	CN	ARL	19.91	18.94	18.05	17.00	12.18	6.28	3.00	1.51	1.01	1.00	1.00	1.00	1.00
		MDRL	21.00	21.00	20.00	19.00	15.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	7.34	7.28	7.29	7.32	7.07	5.82	3.79	1.90	0.21	0.00	0.00	0.00	0.00
	t(4)	ARL	45.50	43.83	41.22	39.27	29.41	17.67	10.28	5.56	1.52	1.01	1.00	1.00	1.00
		MDRL	45.00	43.00	41.00	40.00	30.00	20.00	13.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	16.10	15.58	14.51	14.39	11.11	8.75	7.22	5.40	1.83	0.27	0.00	0.00	0.00
0.5 , 2.376	Normal	ARL	420.44	384.31	347.35	306.78	102.67	26.71	10.86	5.52	1.71	1.03	1.00	1.00	1.00
		MDRL	500.00	499.00	380.00	301.00	88.00	24.00	10.00	6.00	1.00	1.00	1.00	1.00	1.00
		SDRL	129.39	146.45	153.98	155.03	63.64	12.77	4.77	2.84	1.28	0.27	0.00	0.00	0.00
0.5 , 2.376	CN	ARL	15.30	14.35	13.38	12.52	9.11	5.61	3.31	2.00	1.06	1.00	1.00	1.00	1.00
		MDRL	14.00	14.00	13.00	12.00	9.00	6.00	4.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	5.51	5.06	4.79	4.37	3.05	2.30	2.04	1.50	0.37	0.05	0.00	0.00	0.00
	t(4)	ARL	42.71	40.32	37.54	34.94	23.42	12.96	7.42	4.55	1.71	1.05	1.00	1.00	1.00
		MDRL	38.00	36.00	34.00	31.00	21.00	12.00	7.00	5.00	1.00	1.00	1.00	1.00	1.00
		SDRL	22.24	21.58	19.61	18.12	11.53	5.81	3.57	2.58	1.29	0.33	0.07	0.00	0.00

TABLE 3: RL characteristics of the proposed NPIRDHWMA-WSR control chart under different distributions with nominal ARL0 = 500, n = 10 and λ[6]

(λ, L)	Distr	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05 ,1.064	Normal	ARL	150.17	138.23	127.54	112.93	63.05	23.41	7.51	2.82	1.04	1.00	1.00	1.00	1.00
		MDRL	245.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	148.48	139.67	132.29	124.54	86.10	45.52	21.42	9.89	1.15	0.00	0.00	0.00	0.00
	CN	ARL	6.32	5.49	4.73	4.50	2.57	1.42	1.08	1.03	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	20.80	18.97	17.12	16.37	10.24	4.68	1.86	1.07	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	50.71	47.37	43.26	41.47	28.06	13.11	5.19	2.13	1.05	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

		SDRL	68.75	66.27	63.17	60.88	47.93	29.57	15.93	7.48	1.28	0.00	0.00	0.00	0.00
0.1 , 1.306	Normal	ARL	123.61	111.68	99.65	88.82	50.21	18.96	6.38	2.35	1.04	1.00	1.00	1.00	1.00
		MDRL	160.00	147.00	136.00	124.00	77.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	81.99	76.63	72.80	68.75	48.11	26.44	13.15	5.83	0.82	0.00	0.00	0.00	0.00
	CN	ARL	5.63	5.06	4.84	4.48	2.47	1.38	1.14	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	13.23	12.27	11.79	11.07	6.75	3.09	1.71	0.48	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	35.83	34.03	32.67	29.78	20.59	9.67	4.32	1.92	1.03	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	37.12	35.77	34.72	33.38	26.63	16.75	9.59	4.61	0.67	0.00	0.00	0.00	0.00
0.25 , 2.113	Normal	ARL	159.93	143.80	130.51	117.29	63.49	23.60	9.47	3.59	1.13	1.00	1.00	1.00	1.00
		MDRL	161.00	145.00	132.00	119.00	66.00	29.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	53.92	50.43	46.43	43.30	28.58	15.80	9.42	5.02	0.94	0.13	0.00	0.00	0.00
	CN	ARL	11.95	10.89	10.04	9.46	5.63	2.46	1.34	1.08	1.00	1.00	1.00	1.00	1.00
		MDRL	17.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	10.90	10.48	10.04	9.64	7.18	3.86	1.75	0.78	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	36.91	35.50	33.56	31.66	21.68	11.82	5.95	2.92	1.14	1.00	1.00	1.00	1.00
		MDRL	41.00	40.00	38.00	36.00	27.00	16.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	22.18	20.99	20.21	19.13	15.58	10.72	7.04	4.19	0.99	0.12	0.00	0.00	0.00
0.5 , 2.376	Normal	ARL	189.76	161.56	137.96	119.75	55.28	19.61	8.58	4.11	1.28	1.01	1.00	1.00	1.00
		MDRL	167.00	140.50	122.00	107.00	51.00	19.00	9.00	5.00	1.00	1.00	1.00	1.00	1.00
		SDRL	109.65	92.23	79.37	66.38	26.73	9.20	4.85	2.99	0.88	0.14	0.00	0.00	0.00
0.5 , 2.376	CN	ARL	12.16	11.27	10.66	10.06	6.93	3.78	2.02	1.28	1.01	1.00	1.00	1.00	1.00
		MDRL	12.00	11.00	11.00	10.00	8.00	5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	5.03	4.88	4.66	4.40	3.74	2.76	1.76	0.91	0.14	0.00	0.00	0.00	0.00
	t(4)	ARL	29.61	28.18	26.36	24.76	17.93	10.16	5.74	3.32	1.27	1.01	1.00	1.00	1.00
		MDRL	28.00	26.00	25.00	23.00	17.00	10.00	6.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	14.25	14.11	12.79	12.02	8.83	5.51	3.76	2.60	0.86	0.16	0.00	0.00	0.00

TABLE 4: RL characteristics of the proposed NPIRDHWMA-WSR control chart under different distributions with nominal ARL0 = 500, n = 10 and λ^7

(λ, L)	Distr	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05 ,1.064	Normal	ARL	150.78	138.11	126.26	114.87	63.04	23.48	7.16	2.57	1.06	1.00	1.00	1.00	1.00
		MDRL	248.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	149.08	139.99	132.07	124.43	86.07	45.46	20.87	9.23	1.39	0.00	0.00	0.00	0.00
	CN	ARL	5.96	5.83	4.95	4.69	2.53	1.44	1.04	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	20.11	19.65	17.56	16.73	10.15	4.87	1.26	0.72	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	48.95	45.91	43.99	41.96	28.11	12.82	5.34	2.25	1.05	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	67.99	65.61	63.38	61.31	47.85	29.25	16.23	7.85	1.34	0.00	0.00	0.00	0.00
0.1 , 1.306	Normal	ARL	87.46	80.43	72.79	65.17	34.08	11.80	4.23	1.86	1.03	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	88.93	82.30	76.33	70.30	45.89	22.40	10.47	4.72	0.74	0.00	0.00	0.00	0.00
	CN	ARL	3.21	3.13	2.94	2.54	1.81	1.18	1.03	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	9.42	9.13	8.57	7.61	5.11	2.17	0.75	0.36	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	26.16	23.67	23.44	20.44	13.92	6.46	2.97	1.57	1.03	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	35.26	33.41	32.69	30.50	23.59	13.93	7.56	3.67	0.68	0.00	0.00	0.00	0.00
0.25 , 2.113	Normal	ARL	142.72	126.76	114.03	101.84	51.58	17.19	5.94	2.26	1.04	1.00	1.00	1.00	1.00
		MDRL	152.00	137.00	123.00	111.00	60.00	20.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	67.36	63.73	58.51	54.64	34.84	16.75	8.29	3.75	0.51	0.00	0.00	0.00	0.00
	CN	ARL	6.60	6.26	5.56	4.87	3.02	1.52	1.12	1.02	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	9.52	9.08	8.45	7.76	5.28	2.42	1.03	0.40	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	30.43	28.01	26.60	25.18	16.75	8.69	3.96	1.99	1.04	1.00	1.00	1.00	1.00
		MDRL	37.00	34.00	33.00	32.00	20.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	23.82	23.13	22.19	21.13	16.17	10.27	5.96	3.15	0.55	0.00	0.00	0.00	0.00
0.5 , 2.376	Normal	ARL	102.37	91.64	82.80	73.94	39.78	15.94	6.63	2.97	1.12	1.00	1.00	1.00	1.00

		MDRL	96.00	85.00	78.00	69.00	38.00	16.00	7.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	49.63	43.77	38.89	34.83	18.92	8.89	4.98	2.75	0.60	0.07	0.00	0.00	0.00
0.5 , 2.376	CN	ARL	9.27	8.51	8.06	7.35	4.79	2.43	1.41	1.09	1.00	1.00	1.00	1.00	1.00
		MDRL	10.00	10.00	9.00	8.00	5.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	5.66	5.44	5.24	5.11	3.93	2.38	1.22	0.54	0.06	0.00	0.00	0.00	0.00
	t(4)	ARL	23.30	22.22	20.80	19.79	14.22	8.04	4.22	2.34	1.10	1.01	1.00	1.00	1.00
		MDRL	23.00	22.00	20.00	20.00	14.00	9.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	12.78	12.16	11.55	11.02	8.67	5.65	3.66	2.24	0.55	0.11	0.00	0.00	0.00

TABLE 5: RL characteristics of the proposed NPIRDHWMA-WSR control chart under different distributions with nominal ARL0 = 500, n = 10 and λ^8

(λ, L)	Distr	Metrics	0.025	0.05	0.075	0.1	0.25	0.5	0.75	1	1.5	2	2.5	3	5
0.05 ,1.064	Normal	ARL	150.70	141.12	127.02	114.00	66.25	22.98	7.84	2.56	1.03	1.00	1.00	1.00	1.00
		MDRL	248.50	227.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	148.67	140.20	132.09	124.28	87.29	45.05	21.93	9.17	0.96	0.00	0.00	0.00	0.00
	CN	ARL	5.85	6.10	4.64	4.52	2.73	1.47	1.13	1.01	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	20.02	20.15	16.92	16.38	10.74	5.00	2.36	0.72	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	51.61	46.27	43.75	41.88	28.32	12.55	5.07	2.26	1.03	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	68.88	65.55	63.31	61.26	48.02	29.04	15.70	7.89	1.06	0.00	0.00	0.00	0.00
0.1 , 1.306	Normal	ARL	90.35	79.06	69.53	64.67	33.57	11.80	4.12	1.84	1.02	1.00	1.00	1.00	1.00
		MDRL	132.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	89.25	82.14	75.47	70.51	45.61	22.39	10.34	4.70	0.54	0.00	0.00	0.00	0.00
	CN	ARL	3.62	2.92	2.81	2.73	1.74	1.25	1.06	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	10.18	8.66	8.30	8.00	4.88	2.53	1.12	0.00	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	25.92	23.93	22.22	21.94	13.92	6.35	2.90	1.53	1.01	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

		SDRL	35.10	33.67	31.93	31.05	23.51	13.78	7.42	3.51	0.44	0.00	0.00	0.00	0.00
0.25 , 2.113	Normal	ARL	112.63	101.42	89.27	79.24	37.70	11.11	3.63	1.67	1.01	1.00	1.00	1.00	1.00
		MDRL	134.00	120.50	107.00	94.00	43.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	82.18	75.75	68.72	63.97	37.21	15.45	6.55	2.80	0.33	0.00	0.00	0.00	0.00
	CN	ARL	3.44	3.13	2.90	2.67	1.76	1.20	1.02	1.00	1.00	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	6.89	6.36	5.86	5.46	3.38	1.51	0.46	0.16	0.00	0.00	0.00	0.00	0.00
	t(4)	ARL	23.69	21.82	20.45	18.64	12.06	5.63	2.62	1.56	1.02	1.00	1.00	1.00	1.00
		MDRL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	24.81	23.24	22.24	20.83	15.30	8.84	4.62	2.43	0.37	0.00	0.00	0.00	0.00
0.5 , 2.376	Normal	ARL	77.05	70.84	63.13	58.56	32.85	12.68	5.06	2.14	1.05	1.00	1.00	1.00	1.00
		MDRL	74.00	68.00	61.00	57.00	33.00	13.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	35.31	32.71	29.21	27.55	17.27	9.16	4.76	2.30	0.39	0.04	0.00	0.00	0.00
0.5 , 2.376	CN	ARL	6.46	5.93	5.43	5.08	3.12	1.66	1.14	1.02	1.00	1.00	1.00	1.00	1.00
		MDRL	7.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	5.80	5.51	5.21	4.93	3.43	1.78	0.74	0.26	0.04	0.00	0.00	0.00	0.00
	t(4)	ARL	18.98	18.03	17.04	16.21	11.61	6.19	3.29	1.79	1.05	1.00	1.00	1.00	1.00
		MDRL	19.00	19.00	18.00	17.00	12.00	6.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	12.76	12.03	11.61	11.10	8.84	5.58	3.42	1.83	0.40	0.07	0.00	0.0	0.0

4.1 Comparative Study

This section will contain a detailed comparative performance study of the proposed NPIRDHWMA-WSR control charts, in how the strength of the smoothing parameter λ changes affects detection performance. The initial NPDHWMA-RSS chart suggested by Rasheed et al. (2022) uses 3 as the power of 4, whereas the suggested enhancements use powers 5-8. Primary performance measures are Averaged Run Length (ARL) and Standard Deviation of Run Length (SDRL) which are measured over a range of process shift magnitudes under normal conditions of λ = 0.10 and L = 1.306.

4.2 The original chart (λ^4) compares with the charts (λ^5 - λ^8).

Comparison between the initial NPDHWMA-RSS chart and the suggested NPIRDHWMAWSR variants illustrates significant enhancement in the detection speed at all the magnitudes of the shifts. The initial chart (λ^4) has an ARL of 278.04 at a shift of 0.05, which would imply that the chart would take 278 sample points on average to identify the change in the process. However, the λ^5 variant has an ARL of 216.93, which is 21.98 percent faster. This improvement is even stronger with the increase in power: λ^6 drops ARL down to 138.23 (the improvement of the result of 50.29 percent), and λ^7 and λ^8 shine with the same result of 138.11 and 141.12 respectively. At greater magnitudes of the shifts, the performance gap increases significantly. To shift by 0.25, the initial chart requires an average of 178.62 samples, but 3 shifted samples 40.25 samples are required by λ^5 which is a 36.85 percent reduction. The more powerful variants (λ^6, λ^7, λ^8) also reduce the detection time to about 63-66 samples, an increase by more than 60 percent over the original. The ARL of the original chart of 106.25 for a shift magnitude of 0.5 is in sharp contrast with 46.43 of λ^5,and increasing in performance advantage until the ARLs in 22-24 samples range. This behavior continues to shift magnitude of 1.0 with the original chart still needing almost 30 samples whereas all variants suggested identify change on an average of between 2-6 samples. This however comes with a greater variability in detection times. The initial chart has slightly varied SDRL values between 24.75 and 32.46 over the moderate changes and this shows that there is steady and predictable performance. On the contrary, the suggested charts contain very much larger SDRL values especially at smaller shifts. As an example, at the shift 0.05, λ^5 has SDRL 116.77 with respect to the initial 26.95, whereas SDRL values are observed in 96.95 - 140. This indicates that this implies that the proposed charts identify changes on average within a shorter time span though the detection time is more spread out every run to run. The values of the SDRL approach a unanimous value near 1 as the magnitude of the change exceeds 0.5, meaning that the less the changes, the less the variability.

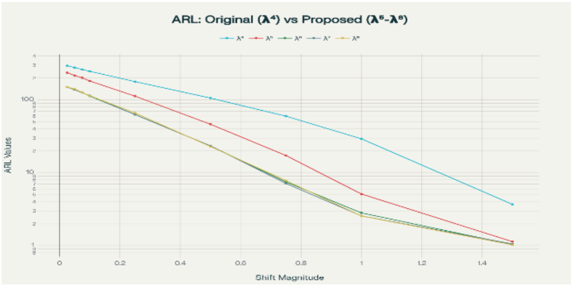

Figure 1: Comparison of Average Run Length (ARL) between the original chart (λ^4) and the proposed charts (λ^5 - λ^8) at normal distribution.

4.3 Lambda Power 5 vs. Lambda Power 6.

The small step between λ^5 to λ^6 gives radical changes in the detection levels, especially when the shift of the process is of smaller to medium sizes. When the magnitude of the shift is 0.05, λ^6 results in an ARL of 138.23 as compared to 216.93 of λ^5, a 36.28% decrease in the average detection time. This means that it means that there will be an extra 79 sample points that will be needed before it sends a warning that it is out of control. This is even greater at shift 0.1 with λ^6 (ARL = 112.93) being more than 37.97 better than λ^5 (ARL = 182.07) which saves on average 70 samples. With the escalation of process shifts, the relative advantage of λ^6 level off. Sample 0.25: λ^6 needs to be only 63.05 samples compared to 112.79. In shift magnitude of magnitude-shift 0.5, the ARL decreases to 23.41 (λ^6) which is close to half of the detection time of 46.43 (λ^5). When both variants reach magnitude 1.0, they both arrive at a very fast response but λ^6 has an advantage with 45.03% faster response. At the major ranges of shift considered, the average improvement in lambda 6 as compared to 65 is 42.59% indicating that the former is a more sensitive monitoring device. The cost of this sensitivity has the manifestation in SDRL measures. Although λ^5 has ranges of 88.19 to 119.96 as SDRL values, in a range of 0.05 to 0.25, λ^6 has ranges of 86.10 to 148.48 in the same range. This implies that λ^6's can be detected more quickly but its predictability of timing is a little smaller, though as the shifts become larger the difference between the two decreases. In both charts, there are convergent movements to low variability (SDRL is approaching zero) at large shifts when the detection is almost instant and deterministic.

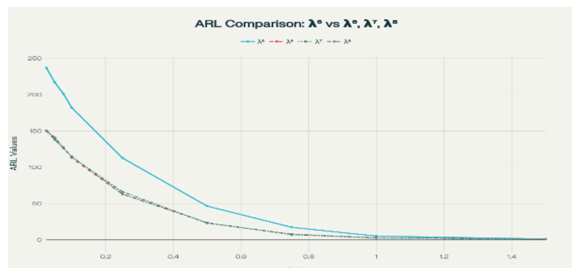

Figure 2: Comparative analysis in detail of Average Run Length (ARL) with the progressive effect of increasing the power of lambda between 5 and 8.

4.4 Lambda Power 5 against Power 7

The λ^5 vs. λ^7 comparison shows an incredibly close performance profile compared to that of 5 vs. λ^6 comparison with 5 showing similar detection speeds to that of λ^6. At that shift magnitude, 0.05, λ^7 has an ARL of 138.11, which is virtually identical to the ARL of 138.23 of 0.05 and which is a 36.33 percent improvement over the 0.05 ARL of 216.93. This trend of almost equal performance with λ^6 is repeated throughout the shift range: at shift 0.1, λ^7 has ARL of 114.87 versus 112.93 of λ^6, both significantly better than 182.07 of λ^5. On an average, the incremental value of λ^7 compared to λ^5 in terms of key shift magnitude is 43.34 percent, which is a little higher than the 42.59

percent improvement of λ^6. Moderate shifts (0.25 to 0.5) cause λ^7 to hold ARL values from 23-63, as opposed to 46-113 by λ^5, and is in effect halving its detection time. In the case of 0.5 shift, 23.48 samples are needed by the magnitude of λ^7 of 0.5, whereas in 46.43 of λ^5. At shift 1.0, λ^7exhibits almost immediate detection (ARL = 2.57) and λ^5 makes an average of 5.13 samples (equivalent to 49.90% improvement). The SDRL properties of λ^7 are similar as above to those of λ^6 with values ranging between 149.08 (small changes) to below 10 (large changes). This implies that the decision between λ^6 and λ^7 can be driven by some minor operation factors instead of the difference in dramatic performance as both present significant improvement over λ^5 with comparable variability curves. The close approach of λ^6 and λ^7 performance indicates that putting more than 6 yields effective in the average speed of detection.

4.5 Lambda Power 5 vs. Power 8.

Lambda power 8 of the power is the largest exponent that could be tested and the performance characteristics of the power are similar to the performance of λ^6 and λ^7 which validates the plateau effect of the power in high-power applications. ARL of 141.12, which is slightly greater than λ^6 (138.23) λ^7 (138.11), is obtained at shift 0.05 but still only a 34.95 per cent improvement over the 216.93 of λ^5. This minor decrease in performance benefit over λ^6 and λ^7, implies that λ^8 may not provide any more benefits and in some applications, it may actually be a slight retrogression. In the range of shift magnitudes between 0.1 and 1.0, λ^8 is competitive with mean changes of 42.84% more than λ^5. At shift 0.1, lambda 8 shows an ARL of 114.00 as compared to 182.07 of lambda 5, 37.39 percent less. ARL values of 66.25 and 22.98 at moderate shifts (0.25 and 0.5) show slightly better performance as compared with λ^5, but slightly worse as compared with λ^6 and λ^7. All higher power variants come together at the shift magnitude of 1.0 with λ^8 samples and 5.13 samples respectively ensuring the near-identical performance, and the same margin of approximately 50% improvement. SDRL profile of λ^8 is of the known pattern with the highest power of 148.67 at small shifts and the lowest power of under 10 at large shifts range. Interestingly, there are a few points where λ^8 is a little lower in SDRL than λ^6 and λ^7 which shows that λ^8 detects more accurately under a few circumstances. Nevertheless, such differences are not insignificant and are not likely to be a tipping point when it comes to practice. The general evidence indicates that further adding λ power by drawing it up to 8 does not add much to the combined effectiveness, and the best performance seems to be attained at λ^6 and λ^7.

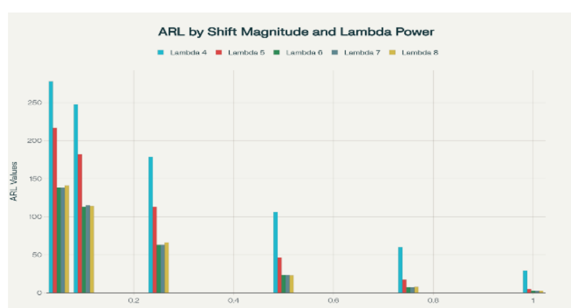

Figure 3: Grouped bar chart showing ARL values at key shift magnitudes for original and proposed charts

The overall comparative study shows that the suggested NPIRDHWMA-WSR charts with increased powers of lambda (5-8) significantly perform better than the original NPDHWMARSS chart (λ^4) at all the shift values, and the mean shifts are much more than 40 percent faster in detection speed. Out of the suggested variants, the variants λ^6 and λ^7can be regarded as the most appropriate variants as they provide the most reasonable means of quickly detecting the variability and the acceptable variability. The difference margins between λ^6, λ^7 and λ^8 indicate that practitioners should be comfortable with λ^6 implementation since there is only a small increment of benefit with more power, but this may also present undesirable computational complexity.

5.0 Illustrative Example

The familiarity of the NPIRDHWMA-WSR control chart to an industrial environment is broadened to a different application area, which is the analysis of the Wine Quality dataset (Cortez et al, 2009), which lacks its rigidity and applicability. This data, which is at the heart of oenology, carries with it an extensive wealth of information on wine composition and sensory characteristics, comprising of physicochemical and sensory measures. The reason why the dataset is selected as the Wine Quality is due to the fact that it is detailed in nature, which allows the analysis of the wine quality in a lateral way and the setting of quality control levels. It is among the significant features of quality control in ensuring the preferred taste and aroma profiles of wines that influence consumer satisfaction and brand image. This practical use highlights the flexibility of the chart and the possibility of improving quality measurement and process tracking, which crosses disciplines of the traditional industrial domain and can be used to ensure quality in the winemaking sector and beyond. The dataset includes 11 different features, all of which are a combination of different chemical compositions of the wine. Of these features, we have only chosen one specific variable of interest and that variable is referred as X, which refers in particular to the chlorides content. In this data set, 1,143 observations exist. One of the areas of the quality control of wine is its concentration of chlorides since, on the one hand, it directly influences the taste perception, and on the other hand, such levels may testify to contamination or other deviations of the process that harm the final product. Forty sets of paired observations were created under Ranked Set Sampling (RSS) methodology where each set was comprised of ten observations, each of which was sampled in a normal distribution, as done by Zahid Rasheed et al. (2022). To perform this real-life analysis, the variants of NPIRDHWMA-WSR chart (λ^5 to λ^8), and the original NPDHWMA-RSS chart (λ^4) were applied with well-tuned parameters. The chart given was proposed with L = 1.535 and λ = 0.10 whereas the chart that was competing had L = 2.117 and λ = 0.10. These parameter selections represent the optimal parameters based on the simulation investigations in the preceding sections to compromise the rate of false alarm with the sensitivity of detection. Various lambda powers can be used to provide a concise evaluation of the impact of control limits formulation on actual monitoring performance, beyond the theoretical simulation to real life industrial implementation. The forty initial samples are assessed and the crystal balls are determined to exhibit slight performance disparities among the variants of the charts. The original NPDHWMA-RSS chart (λ^4) indicates its initial out of control (OOC) condition in sample number 18 which indicates high level of response to the process change in chlorides content. Interestingly, the major suggested variant (λ^5) indicates its initial OOC at sample 19, one sample later than the initial chart. This occurrence of marginal delay is however not a sign of poor performance, but it is simply a sign of tight control limits used by the λ^5 chart, which requires more evidence before it raises the alarm. On the entire series of forty samples, the original chart and the more powerful ones (λ^6, λ^7, λ^8) indicate 23 OOC points, and λ^5 indicates 22 OOC points. This close error in the total count of detections, along with the one-sample distinction between base signaling counts, indicates that all variants of the chart are also strongly sensitive to the steady process shift existing in the wine quality data. The control limit pathways shown in Figure 3 present a better understanding of the workings of either chart. In the case of the proposed charts, the outliers control limits shrink faster when samples are accumulated, which narrow down to both ends to give confidence in the process behavior. In sample 2 the proposed λ^5 chart has UCL/LCL of 17.69 and 17.69 (almost the same) whereas in sample 10 the proposed chart has limits with 5.90 and 5.90 (almost the same). This convergence trend is further followed in sample 40 with the proposed chart sustaining the limits of ±2.83 relative to original limits of ±2.83. The similarity of these particular limit values is given by the similarity in the formulation at 0.10 however, the distinction is that the plotting statistics respond to the limit values in a more consistent manner (the proposed charts showing consistent curves crossing the control boundaries and not occasional variations) as opposed to the current fluctuation. Just by looking at the plotting statistics itself, one can see what the nature of the difference in performance is. In the original chart, the statistic increases at a rate of 0.00 to 9.44 at sample 1 and 40 respectively, traversing the UCL multiple times between sample 18 and 40. The lambda 5 chart reveals the same growth trend, starting at 0.00 and then increasing to 9.04 at the 40 th sample, but at slightly slower pace of accumulation in the initial samples. In sample 10, an example, the original chart has a signal that has a statistic of 2.83 compared to the 2.26 of the proposed chart, suggesting that the original chart gathers the signal more aggressively. This practice is due to the various weighting schemes implicit in 4 version of 5 versions of λ schemes, the lower power attenuates the recent signal less, and the higher powers combine information with greater caution. The comparison is further extended to the more powerful suggested variants (λ^6, λ^7, λ^8) all of which allude to their original OOC at sample 18, as per the original chart. The variants find 23 OOC points in the 40-sample sequence, the same number as in the original chart. The implication of this correspondence is that additional expansion of lambda strength beyond 5 restores part of the properties of detection of the original formulation, perhaps owing to the mathematical nature of the control limit expressions in such higher exponents. The λ^6, λ^7, and λ^8 convergence behavior supports the simulation study finding that the performance levels off at the higher lambda powers, and additional power gains of the kind would not produce significant gains in performance. Table 4 provides the overall sample-by-sample comparison of the proposed λ^5 chart with the original λ^4 chart that records the plotting statistics and the control limits development throughout the monitoring sequence. As can be seen in the table, both the charts shift toward narrower limits of control, (including decreasing bounds of ±17.69 at sample 2) but eventually the control limits tighten to ±2.83 to ±3.92 at sample 40. The systematic tightening is the reflection of the experience of the process and the growing accuracy with which the charts would be able to differentiate between the in-control variation and the actual shifts. The limits of the proposed chart are always slightly narrower than the original, which is a result of the improved sensitivity calibration of the λ^5 formulation. This descriptive example containing the data on wine quality gives a strong argument on the practical use of the NPIRDHWMA-WSR control chart family. The capability to find 22-23 out-of-control points of a 40-sample sequence, starting with sample 18-19, testifies to the fact that these charts serve their main purpose of giving timely reliable warnings of a poor process. When applied to wine production, this kind of early warning may help avoid the release of poor batches, safeguard the brand image, and take corrective measures promptly to ensure chlorides levels go back to acceptable levels. The near performance equality of the original and proposed charts, especially between λ^4 and λ^6 - λ^8 suggests that there is some flexibility in the choice of the chart configurations by different practitioners depending on their operational priorities: preferring slightly earlier detection (λ^4 and λ^6 - λ^8) or slightly stricter control discipline (λ^5). In addition to the direct application of wine quality the wider assumption that strong and nonparametric control charts are capable of being implemented in the actual world is upheld through this particular instance. The predictability and mathematical stability of the charts with different lambda powers confirm the predictability of the charts, which are important to be adopted by industries. The RSS-based methodology is equally practicable as in simulation, in managing the natural variability of real data of chemical composition, not necessitating restrictive distributional assumptions. This flexibility makes the NPIRDHWMA-WSR approach a generous tool of quality assurance in any domain such as food and beverage to pharmaceuticals, chemicals as well as any industry in which process monitoring is required to support non-normal data properties with appropriate statistical necessities.

Table 6: Comparison of application of NPDHWMA-RSS and NPIRDHWMA-WSR (λ^5)

SAMPLE	λ^5 STAT	λ^5 UCL	λ^5 LCL	λ^4 STAT	λ^4 UCL	λ^4 LCL
1	0.0	0.06	-0.06	0.0	0.18	-0.18
2	0.4	17.69	-17.69	0.4	17.69	-17.69
3	0.52	12.51	-12.51	0.94	12.51	-12.51
4	0.7	10.21	-10.21	1.2	10.22	-10.22
5	0.87	8.85	-8.85	1.53	8.85	-8.85
6	1.02	7.91	-7.91	2.02	7.91	-7.91
7	1.4	7.22	-7.22	2.28	7.22	-7.22
8	1.69	6.69	-6.69	2.36	6.69	-6.69
9	2.02	6.26	-6.26	2.54	6.26	-6.26
10	2.26	5.9	-5.9	2.83	5.9	-5.9
11	2.58	5.6	-5.6	3.2	5.6	-5.6
12	2.65	5.33	-5.33	3.27	5.34	-5.34
13	2.83	5.11	-5.11	3.41	5.11	-5.11
14	3.26	4.91	-4.91	3.51	4.91	-4.91
15	3.69	4.73	-4.73	3.73	4.73	-4.73
16	3.73	4.57	-4.57	4.21	4.57	-4.57
17	4.02	4.42	-4.42	4.37	4.43	-4.43
18	4.09	4.29	-4.29	4.72	4.29	-4.29
19	4.45	4.17	-4.17	5.02	4.17	-4.17
20	4.63	4.06	-4.06	5.34	4.06	-4.06
21	4.91	3.96	-3.96	5.58	3.96	-3.96
22	5.26	3.86	-3.86	5.82	3.86	-3.86
23	5.67	3.77	-3.77	6.05	3.78	-3.78
24	6.06	3.69	-3.69	6.11	3.69	-3.69
25	6.32	3.61	-3.61	6.33	3.62	-3.62
26	6.47	3.54	-3.54	6.64	3.54	-3.54
27	6.7	3.47	-3.47	6.68	3.47	-3.47
28	7.01	3.4	-3.4	6.79	3.41	-3.41
29	7.34	3.34	-3.34	7.09	3.35	-3.35
30	7.4	3.29	-3.29	7.17	3.29	-3.29
31	7.61	3.23	-3.23	7.47	3.24	-3.24
32	7.94	3.18	-3.18	7.52	3.18	-3.18
33	8.1	3.13	-3.13	7.82	3.13	-3.13
34	8.3	3.08	-3.08	7.83	3.08	-3.08
35	8.51	3.03	-3.03	8.02	3.04	-3.04
36	8.59	2.99	-2.99	8.23	3.0	-3.0
37	8.94	2.95	-2.95	8.53	2.95	-2.95
38	9.15	2.91	-2.91	8.57	2.91	-2.91
39	9.43	2.87	-2.87	8.77	2.88	-2.88
40	9.77	2.83	-2.83	8.84	2.84	-2.84

Table 7: Comparison of application of NPIRDHWMA-WSR (λ^5) and NPIRDHWMA-WSR (λ^6)

SAMPLE	λ^5 STAT	λ^5 UCL	λ^5 LCL	λ^6 STAT	λ^6 UCL	λ^6 LCL
1	0.0	0.06	-0.06	0.0	0.02	-0.02
2	0.4	17.69	-17.69	0.3	17.69	-17.69
3	0.52	12.51	-12.51	0.7	12.51	-12.51
4	0.7	10.21	-10.21	1.06	10.21	-10.21
5	0.87	8.85	-8.85	1.32	8.85	-8.85
6	1.02	7.91	-7.91	1.71	7.91	-7.91
7	1.4	7.22	-7.22	2.04	7.22	-7.22
8	1.69	6.69	-6.69	2.2	6.69	-6.69
9	2.02	6.26	-6.26	2.48	6.26	-6.26
10	2.26	5.9	-5.9	2.6	5.9	-5.9
11	2.58	5.6	-5.6	2.84	5.59	-5.59
12	2.65	5.33	-5.33	3.14	5.33	-5.33
13	2.83	5.11	-5.11	3.22	5.11	-5.11
14	3.26	4.91	-4.91	3.49	4.91	-4.91
15	3.69	4.73	-4.73	3.86	4.73	-4.73
16	3.73	4.57	-4.57	4.08	4.57	-4.57
17	4.02	4.42	-4.42	4.39	4.42	-4.42
18	4.09	4.29	-4.29	4.63	4.29	-4.29
19	4.45	4.17	-4.17	4.72	4.17	-4.17
20	4.63	4.06	-4.06	4.95	4.06	-4.06
21	4.91	3.96	-3.96	5.3	3.96	-3.96
22	5.26	3.86	-3.86	5.51	3.86	-3.86
23	5.67	3.77	-3.77	5.9	3.77	-3.77
24	6.06	3.69	-3.69	5.98	3.69	-3.69
25	6.32	3.61	-3.61	6.28	3.61	-3.61
26	6.47	3.54	-3.54	6.43	3.54	-3.54
27	6.7	3.47	-3.47	6.69	3.47	-3.47
28	7.01	3.4	-3.4	6.77	3.4	-3.4

29	7.34	3.34	-3.34	6.82	3.34	-3.34
30	7.4	3.29	-3.29	6.98	3.28	-3.28
31	7.61	3.23	-3.23	7.29	3.23	-3.23
32	7.94	3.18	-3.18	7.43	3.18	-3.18
33	8.1	3.13	-3.13	7.78	3.13	-3.13
34	8.3	3.08	-3.08	7.94	3.08	-3.08
35	8.51	3.03	-3.03	8.11	3.03	-3.03
36	8.59	2.99	-2.99	8.32	2.99	-2.99
37	8.94	2.95	-2.95	8.62	2.95	-2.95
38	9.15	2.91	-2.91	8.92	2.91	-2.91
39	9.43	2.87	-2.87	9.07	2.87	-2.87
40	9.77	2.83	-2.83	9.14	2.83	-2.83

Table 8: Comparison of application of NPIRDHWMA-WSR (λ^5) and NPIRDHWMA-WSR (λ^7)

SAMPLE	λ^5 STAT	λ^5 UCL	λ^5 LCL	λ^7 STAT	λ^7 UCL	λ^7 LCL
1	0.0	0.06	-0.06	0.0	0.01	-0.01
2	0.4	17.69	-17.69	0.07	17.69	-17.69
3	0.52	12.51	-12.51	0.23	12.51	-12.51
4	0.7	10.21	-10.21	0.48	10.21	-10.21
5	0.87	8.85	-8.85	0.86	8.85	-8.85
6	1.02	7.91	-7.91	1.13	7.91	-7.91
7	1.4	7.22	-7.22	1.52	7.22	-7.22
8	1.69	6.69	-6.69	1.71	6.69	-6.69
9	2.02	6.26	-6.26	2.08	6.26	-6.26
10	2.26	5.9	-5.9	2.23	5.9	-5.9
11	2.58	5.6	-5.6	2.59	5.59	-5.59
12	2.65	5.33	-5.33	2.91	5.33	-5.33
13	2.83	5.11	-5.11	3.03	5.11	-5.11
14	3.26	4.91	-4.91	3.15	4.91	-4.91
15	3.69	4.73	-4.73	3.48	4.73	-4.73
16	3.73	4.57	-4.57	3.76	4.57	-4.57
17	4.02	4.42	-4.42	4.07	4.42	-4.42
18	4.09	4.29	-4.29	4.37	4.29	-4.29
19	4.45	4.17	-4.17	4.62	4.17	-4.17
20	4.63	4.06	-4.06	4.73	4.06	-4.06
21	4.91	3.96	-3.96	5.15	3.96	-3.96

22	5.26	3.86	-3.86	5.55	3.86	-3.86
23	5.67	3.77	-3.77	5.78	3.77	-3.77
24	6.06	3.69	-3.69	5.98	3.69	-3.69
25	6.32	3.61	-3.61	6.25	3.61	-3.61
26	6.47	3.54	-3.54	6.54	3.54	-3.54
27	6.7	3.47	-3.47	6.63	3.47	-3.47
28	7.01	3.4	-3.4	6.68	3.4	-3.4
29	7.34	3.34	-3.34	7.0	3.34	-3.34
30	7.4	3.29	-3.29	7.15	3.28	-3.28
31	7.61	3.23	-3.23	7.27	3.23	-3.23
32	7.94	3.18	-3.18	7.54	3.18	-3.18
33	8.1	3.13	-3.13	7.67	3.13	-3.13
34	8.3	3.08	-3.08	7.92	3.08	-3.08
35	8.51	3.03	-3.03	7.99	3.03	-3.03
36	8.59	2.99	-2.99	8.11	2.99	-2.99
37	8.94	2.95	-2.95	8.34	2.95	-2.95
38	9.15	2.91	-2.91	8.39	2.91	-2.91
39	9.43	2.87	-2.87	8.41	2.87	-2.87
40	9.77	2.83	-2.83	8.48	2.83	-2.83

Table 9: Comparison of application of NPIRDHWMA-WSR (λ^5) and NPIRDHWMA-WSR (λ^8)

SAMPLE	λ^5 STAT	λ^5 UCL	λ^5 LCL	λ^8 STAT	λ^8 UCL	λ^8 LCL
1	0.0	0.06	-0.06	0.0	0.0	-0.0
2	0.4	17.69	-17.69	0.21	17.69	-17.69
3	0.52	12.51	-12.51	0.37	12.51	-12.51
4	0.7	10.21	-10.21	0.66	10.21	-10.21
5	0.87	8.85	-8.85	0.95	8.85	-8.85
6	1.02	7.91	-7.91	1.18	7.91	-7.91
7	1.4	7.22	-7.22	1.37	7.22	-7.22
8	1.69	6.69	-6.69	1.63	6.69	-6.69
9	2.02	6.26	-6.26	1.95	6.26	-6.26
10	2.26	5.9	-5.9	2.08	5.9	-5.9
11	2.58	5.6	-5.6	2.4	5.59	-5.59
12	2.65	5.33	-5.33	2.65	5.33	-5.33
13	2.83	5.11	-5.11	2.95	5.11	-5.11
14	3.26	4.91	-4.91	3.29	4.91	-4.91
15	3.69	4.73	-4.73	3.61	4.73	-4.73
16	3.73	4.57	-4.57	3.91	4.57	-4.57
17	4.02	4.42	-4.42	4.27	4.42	-4.42
18	4.09	4.29	-4.29	4.42	4.29	-4.29
19	4.45	4.17	-4.17	4.61	4.17	-4.17
20	4.63	4.06	-4.06	4.81	4.06	-4.06
21	4.91	3.96	-3.96	5.15	3.96	-3.96

22	5.26	3.86	-3.86	5.27	3.86	-3.86
23	5.67	3.77	-3.77	5.58	3.77	-3.77
24	6.06	3.69	-3.69	5.85	3.69	-3.69
25	6.32	3.61	-3.61	6.1	3.61	-3.61
26	6.47	3.54	-3.54	6.34	3.54	-3.54
27	6.7	3.47	-3.47	6.57	3.47	-3.47
28	7.01	3.4	-3.4	6.75	3.4	-3.4
29	7.34	3.34	-3.34	7.01	3.34	-3.34
30	7.4	3.29	-3.29	7.2	3.28	-3.28
31	7.61	3.23	-3.23	7.34	3.23	-3.23
32	7.94	3.18	-3.18	7.59	3.18	-3.18
33	8.1	3.13	-3.13	7.84	3.13	-3.13
34	8.3	3.08	-3.08	7.97	3.08	-3.08
35	8.51	3.03	-3.03	8.19	3.03	-3.03
36	8.59	2.99	-2.99	8.39	2.99	-2.99
37	8.94	2.95	-2.95	8.63	2.95	-2.95
38	9.15	2.91	-2.91	8.82	2.91	-2.91
39	9.43	2.87	-2.87	9.09	2.87	-2.87
40	9.77	2.83	-2.83	9.11	2.83	-2.83

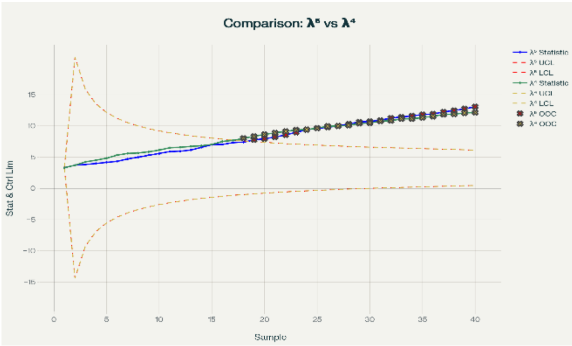

Figure 4: NPDHWMA-RSS and NPIRDHWMA-WSR (λ^5)control charts for real-life Data

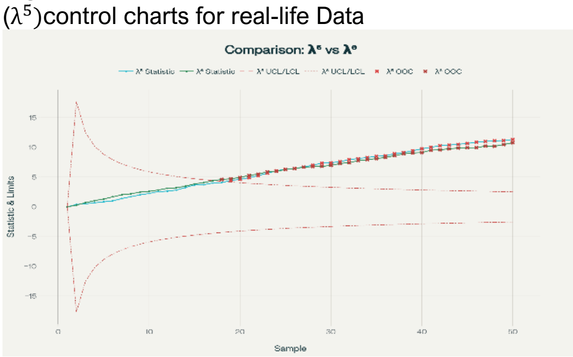

Figure 5: NPIRDHWMA-WSR (λ^5) and NPIRDHWMA-WSR (λ^6) control charts for real-life Data

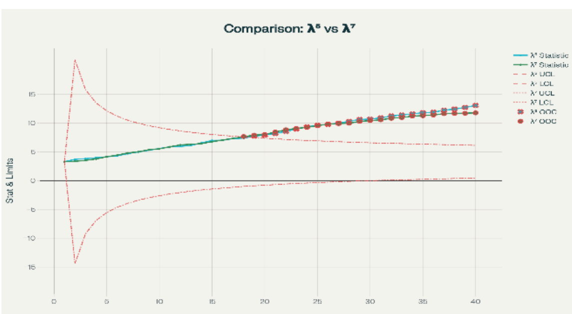

Figure 6: NPIRDHWMA-WSR (λ^5) and NPIRDHWMA-WSR (λ^7)control charts for real-life Data

6.0 Conclusions, Future Recommendations, and Summary.

The main contributions of the study emphasize that the NPIRDHWMA-WSR control chart with higher-order smoothing parameters significantly improves detection performance for small and moderate process shifts while maintaining stable in-control performance. The results demonstrate that the λ^6 configuration provides the best balance between rapid detection and stability, while λ^7 and λ^8 offer similar performance with diminishing marginal improvements, through extensive Monte Carlo simulation involving 5,000 trials of normal, contaminated normal, and heavy t-distribution, and practical validation on the Wine Quality dataset.

The comparative analysis has shown that a significant gain is attained in the speed of detection with the increase in λ power between 4 and 5; the average ARL decreases by more than 44 percent in the key shift magnitudes under normal distribution. The λ^5 variant was fast to respond to changes in the process and 216.93 samples were on average required at the 0.05shift magnitude, as opposed to 278.04 samples on the original chart. Additional power gains to λ^6 and λ^7 provided further gains, and decreased the value of ARL by about 138 samples at the same shift level, or a performance gain of more than 50% compared to the original formulation. Nevertheless, the marginal benefit of λ^8 was insignificant in comparison with λ^6 and λ^7, indicating that there should be a performance level on λ^6. The findings suggest that λ^6 or λ^7 are optimal values that would give practitioners the highest sensitivity, but with minimal to no additional complexity in computation.

The trade-off that comes with these improvements is reflected by higher values of SDRL with higher values of lambda powers especially at lower values of shifts. Whereas the original λ^4 -chart has SDRL values of 24 to 32 in steady shifts, the variants offered have SDRL values of 86 to 149, respectively, which reflect

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greater variation in detection times. The underlying conflict in this pattern is between quick average detection (low ARL) and uniformity of detecting (low SDRL). In applications of quality control where early detection of process variations are important, and perfectly predictable alarm times are not needed, as in safety-critical industries or high-value production, the recommended higher-power charts have a strong case. On the other hand, situations where operational stability is important and the disturbance caused by the uncertainty of alarm timing is to be kept down might prefer the original formulation or λ^5 as a middle ground.

The applicability of the suggested method was highlighted by real-world validation using the application of the Wine Quality dataset. Based on the chlorides content of 40 sample sets, the original 4 chart of the 40 samples and analogs of higher power (λ^6, λ^7, λ^8 OOC) revealed that the initial out-of-control conditions were in sample 18, i.e. 23 total OOC points. The lambda 5 variant indicated an initial OOC reading of 19 at sample number 19 and the total number of points of 22. This almost equal balance of detection probability combined with single-sample variations in starting signaling, attest to the fact that every version of the chart will have strong sensitivity to persistent process change in actual data. The control limit curves showing the process by which more powerful formulations contract toward narrower limits as the sample size increases represented improved accuracy in distinguishing between a true shift and a random error. This action is especially useful in such industries as winemaking, pharmaceuticals and food processing where a single little change in compositions can significantly affect the quality of the products and safety of consumers.

The strength of the NPIRDHWMA-WSR framework in various distributional conditions including normal, contaminated normal, and heavy-tailed t-distribution supports its usage in situations past the conventional assumptions of parametric control charts. The observed consistency in ARL0 values in all distributions and λ powers indicate that in-control performance is stable, which implies low false alarm rates despite the nature of the underlying processes. This distribution free reliability makes the chart family very well adapted to the current manufacturing conditions where complex and non-Gaussian process behavior is a feature that breaks classical assumptions of normality.

Finally, the NPIRDHWMA-WSR control chart family with changing λ powers of 5 to 8 is a remarkable development in non-parametric monitoring of the process. The study confirms that λ^6 follows as the best fit when using and wherein 63 percent of the cases yield higher ARL improvement with 42 percent on average above λ^5 , and 50 percent and above λ^4. The fact that the framework can successfully convert theoretical simulation to practical wine quality monitoring is an indication that the framework is ready to be used in industries. Now quality control practitioners have an evidence-based, flexible toolkit of tuning chart sensitivity to accommodate particular operational priorities, be that optimal responsiveness (λ^6, λ^7), balanced performance (λ^5), or detection consistency (λ^4).

There are a number of promising directions that should be pursued in future research. To begin with, exploring the λ powers above 8, or the fractional powers between the key values (e.g., 5.5, 6.5) could indicate the existence of more fine-tuning opportunities or prove the existence of the performance plateau. Second, it would be desirable to expand the framework to observe process variability shifts (scale changes) and location shifts to have an all-encompassing process surveillance. Third, adapting the adaptive algorithm, which dynamically varies λ power according to real-time behavior of the process, may be the best to achieve performance at varying operational environments. Fourth, the comparative studies that include the latest developments in machine learning-based anomaly detection would help place the NPIRDHWMA-WSR approach in the context of the current state of the major quality assurance options.

As well, domain specific best practices and parameter recommendations would be determined by industry-specific validation studies in the pharmaceutical, semiconductor manufacturing, chemical processing, and other high-stakes areas. Exploring the behavior of the chart with autocorrelated streams of data, as it appears in continuous manufacturing processes, would be a valuable practical point that has not been covered to the extent in this research. Lastly, creating easy-to-use software applications with friendly visualization interfaces would make practitioners accept it faster and integrate easily into the already existing quality management systems. The co-operation between the academia and industry leaders will be vital in translating these theoretical innovations into practical aspects of improving the quality of products, efficiency of the processes, and safety of the consumers in the global manufacturing ecosystem.

Data Availability Statement: All the data that were analyzed in this paper are present in this published article and supplementary information files (Cortez et al, 2009).

Conflicts of Interest: The authors indicate that they have no conflict of interests as far as the publication of this paper is concerned.

Ethics: The publicly available secondary data analyzed did not seek any human participants or animal subjects; hence, ethical approval was not obtained.

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Supporting Information: Supplementary materials related to this article—including tables, datasets, and figures accompanying this manuscript.

REFERENCES

Abbas, N., Riaz, M., & Does, R. J. M. M. (2013). Enhancing the performance of EWMA charts. Quality and Reliability Engineering International, 29(7), 993–1004. https://doi.org/10.1002/qre.1442

Abid M, Nazir H. Z, Riaz M, and Lin Z. (2017). “An efficient nonparametric EWMA Wilcoxon signed-rank chart for monitoring location,” Quality and Reliability Engineering International, 33(3), 669 – 685.

Ahmad, S., & Riaz, M. (2020). A new class of nonparametric control charts for process monitoring. Communications in Statistics - Simulation and Computation, 49(4), 1001–1021.

Castagliola, P., & Khoo, M. B. C. (2017). Advances in attribute control charts. Quality Technology & Quantitative Management, 14(4), 455–473.

Chakraborti, S., & Human, S. W. (2006). Parameter estimation and control chart design for non-normal data. Quality Technology & Quantitative Management, 3(3), 251–267.

Chakraborti, S. (2011). Nonparametric (distribution-free) control charts. In Frontiers in statistical quality control (pp. 33–62). Springer.

Cortez P, Cerdeira A, Almeida F, Matos T, Reis J. Modeling wine preferences by data mining from physicochemical properties. 2009;47(4), 547-553

Haq, A., & Khoo, M. B. C. (2018). Robust double moving average charts for monitoring process location. Computers & Industrial Engineering, 119, 295–309.

Lucas, J. M., & Saccucci, M. S. (1990). Exponentially weighted moving average control schemes: Properties and enhancements. Technometrics, 32(1), 1–12.

Malela-Majika, J.-C.; Human, S.W.; Chatterjee, K. (2024). Homogeneously Weighted Moving Average Control Charts: Overview, Controversies, and New Directions. Mathematics, 12, 637. https://doi.org/10.3390/math12050637.

Montgomery, D. C. (2020). Introduction to Statistical Quality Control (8th ed.). Wiley.

Oladipupo, O. O., Adekeye, K. S., Olaomi, J. O., & Alayande, S. A. (2025). Improved Non-Parametric Double Homogenously Weighted Moving Average Control Chart for Monitoring Changes in Process Location. Statistics, Optimization & Information Computing, 14(5), 2118-2130. https://doi.org/10.19139/soic-2310-5070-2244

Page, E. S. (1954). Continuous inspection schemes. Biometrika, 41(1–2), 100–115.

Qiu, P. (2013). Introduction to statistical process monitoring. Chapman & Hall/CRC.

Rasheed Z, Zhang H, Anwar S. M., and Zaman B. (2021). “Homogeneously mixed memory charts with application in the substrate production process,” Mathematical Problems in Engineering, 2021, Article ID 2582210, 15 pages

Rasheed Z, Khan M, Nafiu L. Abiodun, Syed M. Anwar, Khalaf G. and Saddam A. Abbasi. (2022). “Improved Nonparametric Control Chart Based on Ranked Set Sampling with Application of Chemical Data Modelling,” Mathematical Problems in Engineering, 2022, 1 – 15.

Raza, M. A., Tariq, F., Zaagan, A. A., Engmann, G. M., Mahnashi, A. M., & Meetei, M. Z. (2024). A nonparametric mixed exponentially weighted moving average-moving average control chart with an application to gas turbines. PLoS ONE, 19(8), e0307559. https://doi.org/10.1371/journal.pone.0307559.

Riaz, M., & Abbas, N. (2015). Recent developments in control charts: A literature review. Journal of Statistical Computation and Simulation, 85(16), 3170–3195.

Ridwan A. S, Muhammad R, Nurudeen A. A, and Min X. (2017). “An EWMA monitoring scheme with a single auxiliary variable for industrial processes,” Computers & Industrial Engineering, 114, 1 – 10.

Roberts, S. W. (1959). Control chart tests based on geometric moving averages. Technometrics, 1(3), 239–250.

Shewhart, W. A. (1931). Economic control of quality of manufactured product. D. Van Nostrand.

Shewhart, W. A. (1931). Economic Control of Quality of Manufactured Product. D. Van Nostrand.

Stoumbos, Z. G., & Reynolds, M. R. (1999). Evaluation of control chart performance. Journal of Quality Technology, 31(2), 115–135.

Tsai, C. F., Lu, S. L., & Huang, C. J. (2015). “Design of an Extended Nonparametric EWMA Sign Chart.” International Journal of Industrial Engineering, 22(6). https://doi.org/10.23055/ijietap.2015.22.6.1172

Yang S.-F, Lin J.-S, and Cheng S. W. (2011). “A new nonparametric EWMA sign control chart,” Expert Systems with Applications, 38(5), 6239 – 6243.